TURBULENT BOUNDARY LAYER ON A CHEMICALLY ACTIVE SURFACE WITH A NEGATIVE PRESSURE GRADIENT

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An integral method of flow analysis is discussed for a turbulent boundary layer on an ablating surface of plane and axisymmetric bodies. The results of computations are given for ablation of the surfaces of a plate and a sphere of a material similar in composition to textolite.

Many investigations have been devoted to analyzing the ablation and damage of thermal-insulation coatings. The most reliable results in this realm have been obtained for laminar flow in a boundary layer; much less work has been done on the more important practical case of turbulent flow.

One of the first studies of a turbulent boundary layer on a chemically active surface was reported by Denison [1], who generalized the semiempirical theory of Prandtl to the case of a compressible fluid. Results of calculations of the ablation of a graphite surface are given as an example.

In later investigations [2-4] consideration has been given to a frozen turbulent boundary layer on the ablating surface of a plate with regard for differences in the diffusive properties of the components. Power-law velocity profiles in a boundary layer are used in [2] for the analysis of friction. The application of the proposed method is illustrated in the combustion of a graphite surface in a gas flow comprising a mixture of oxygen and nitrogen. In [3, 4] friction is analyzed on the basis of the semiempirical theory of von Kármán. The problem has also been solved in the local similarity approximation for a turbulent boundary layer [5]. The ablation of textolite [3] and asbotextolite [4] is considered as an example.

We now propose a method for the analysis of plane or axisymmetric flow in a turbulent boundary layer on a chemically active surface in the presence of a negative or zero pressure gradient. We adopt a two-layer model of the turbulent boundary layer. We consider the flow in the layer to be frozen-in. The turbulent analogs of the Prandtl and Schmidt numbers are taken to be unity. We neglect the effects of pressure and temperature diffusion as well as radiative heat transfer from the gas to the surface.

The equations for a multicomponent turbulent boundary layer in the case of a gaseous mixture of N chemical components flowing past an axisymmetric body of revolution have the following form with regard for the stated assumptions [5]:

$$\frac{\partial}{\partial x}(\rho u r_\omega^\beta) + \frac{\partial}{\partial y}(\rho v r_\omega^\beta) = 0, \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial \tau}{\partial y}, \tag{2}$$

$$\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} = -\frac{\partial I_i}{\partial y} \quad (i = 1, 2, \ldots, N-1), \tag{3}$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = (\mu + \varepsilon)\left(\frac{\partial u}{\partial y}\right)^2 + u \frac{dp}{dx} + \frac{\partial}{\partial y}\left[(\lambda + \lambda_\tau)\frac{\partial T}{\partial y}\right] - \frac{\partial T}{\partial y}\sum_{i=1}^{N} c_{pi} I_i, \tag{4}$$

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$$\sum_{i=1}^{N} c_i = 1,\ p = \rho \frac{R}{m} T,\ m = \left(\sum_{i=1}^{N} \frac{c_i}{m_i} \right)^{-1},\ c_p = \sum_{i=1}^{N} c_{pi} c_i, \tag{5}$$

$$\tau = (\mu + \varepsilon) \frac{\partial u}{\partial y},\ I_i = -\rho (D_i + D_T) \frac{\partial c_i}{\partial y}. \tag{6}$$

Here $\beta = 0$ for plane and $\beta = 1$ for axisymmetric flow in the boundary layer, and D_i is the effective diffusivity of the i-th component according to equations given in [6]. The boundary conditions at the outer edge of the boundary layer have the form

$$u \to u_e,\ c_i \to c_{ie},\ T \to T_e \quad \text{as} \quad y \to \infty, \tag{7}$$

and the gas at the outer edge is assumed to have equilibrium composition.

On the surface of the body we have

$$u = 0 \quad \text{at} \quad y = 0. \tag{8}$$

The following boundary conditions on the ablating surface are determined by solving simultaneously the equilibrium equations of the chemical reactions while conserving the elements and balance of energy on the surface:

$$K_{pi} = \prod_{j=1}^{N} \left(\frac{c_{jw}}{m_j} mp \right)^{\nu''_{ij} - \nu'_{ij}},\ i = 1, 2, \ldots, N - l, \tag{9}$$

$$\sum_{i=1}^{N} r_{ki} [(\rho v)_w (c_{iw} - c_i^{(1)}) + I_i] = 0 \quad (k = 1, 2, \ldots, l-1),\ \sum_{i=1}^{N} c_{iw} = 1, \tag{10}$$

$$(\rho v)_w (h_w - h^{(1)}) - \left(\lambda \frac{\partial T}{\partial y} \right)_w + \sum_{i=1}^{N} h_{iw} I_{iw} + \gamma \sigma T_w^4 = (\rho v)_w (h_{sw} - h_{-\infty} + \Delta). \tag{11}$$

Here K_{pi} denotes the equilibrium constants of the chemical reactions taking place on the surface. To close the system (9)-(11) we invoke the ablation kinetic equation in the form $(\rho v)_w = f(T_w)$ from [7].

In the diffusion equations (3) and energy equations (4) we transform to the modified Crocco variables [3]

$$\xi = x,\ z = 1 + Bu/u_e \tag{12}$$

and integrate these equations in the local similarity approximation, using a linear distribution function for the frictional stress. As a result we obtain the well-known relations between the concentration, temperature, and velocity profiles in the laminar sublayer and turbulent core of a boundary layer [3, 5, 8].

In the laminar sublayer ($1 \leq z \leq z_L$)

$$c_i(z) = L_i + (c_{iw} - L_i) \exp \left(\int_1^z \mathrm{Sc}_i \frac{dz}{z} \right), \tag{13}$$

$$T(z) = \frac{1}{W(z)} \left\{ T_w + [1 - W(z)] \frac{\delta}{c_p^0} + Q(z) \frac{u_e^2}{2c_{pe}} \right\}, \tag{14}$$

and in the turbulent core ($z_L \leq z \leq 1 + B$)

$$c_i(z) = \frac{z}{z_e} c_{ie} + L_i \left(1 - \frac{z}{z_e} \right), \tag{15}$$

$$T(z)=\frac{1}{P(z)}\left\{T_e+[1-P(z)]\frac{\delta}{c_p^0}+R(z)\frac{u_e^2}{2c_{pe}}\right\}. \tag{16}$$

Here we have introduced the notation

$$L_i=c_{iw}+z_L(c_{ie}-c_{iw})\left[z_L-(1+B)\exp\left(\int_1^{z_L}\mathrm{Sc}_i\frac{dz}{z}\right)\right]^{-1},$$

$$W(z)=\exp\left(-\int_1^z\frac{c_p^0}{c_p}\,\mathrm{Pr}\,\frac{dz}{z}\right),\quad R(z)=\frac{z_e}{B^2}\left(z_e+\frac{1}{z_e}-z-\frac{1}{z}\right),$$

$$P(z)=1+\frac{c_p^0}{c_p}\left(\frac{z_e}{z}-1\right),\quad Q(z)=\frac{1}{B^2}\cdot\frac{c_{pe}}{c_p^0}\left[(z^2-1)W-2\int_1^z Wz\,dz\right], \tag{17}$$

$$\frac{\delta}{c_p^0}=\frac{P_L}{W_L-P_L}(T_e-T_w)+\frac{R_L-Q_L}{W_L-P_L}\cdot\frac{u_e^2}{2c_{pe}}-T_w,\quad c_p^0=\sum_{i=1}^N c_{pi}L_i.$$

To compute the molecular thermal conductivity λ and viscosity μ in the laminar sublayer we use the familiar relations of molecular kinetic theory [9].

We compute the coefficient of surface friction by the method of successive approximations [5], using the following relations deduced from the momentum integral equation:

$$c_f=\left(\frac{0{,}245K}{0{,}3+E}\right)^2,\quad K=\frac{1}{B}\int_0^{1+B}\sqrt{\frac{\omega}{z}}\,dz,$$

$$E=\frac{\exp S}{\mu_w(1+B)}\left[\frac{1}{2}\int_{x_t}^x c_f\rho_e u_e(1+B)\exp S\,dx+(\rho_e u_e\delta^{**})_t\right], \tag{18}$$

$$\omega=\frac{\rho}{\rho_e},\quad S=\int_{x_t}^x\left[\frac{(1+H)}{u_e}\frac{du_e}{dx}+\frac{1}{r_w^\beta}\frac{dr_w^\beta}{dx}\right]dx.$$

Unlike [5], here we apply the method of iterations to compute the friction in the presence of boundary-layer injection, so that relations (18) include the injection parameter B. The asymptotic expansions given in [5] for the integral thicknesses δ^* and δ^{**} are also somewhat altered as a result of injection:

$$\delta^*=F\left[1+B\omega_e'-B\sqrt{(1+B)\omega_w}\left(B\omega_e''+\frac{0{,}5+2B}{1+B}\omega_e'-\right.\right.$$
$$\left.\left.-\frac{3}{2}B\omega_e'^2+\frac{3}{2}\frac{1}{1+B}\right)\frac{1}{\varkappa\zeta}\right],$$

$$\delta^{**}=F\left[1-\sqrt{(1+B)\omega_w}\left(\frac{3{,}5B+2}{1+B}+\frac{1}{2}B\omega_e'\right)\frac{1}{\varkappa\zeta}\right], \tag{19}$$

$$F=\frac{\alpha}{\varkappa}\cdot\frac{\mu_w(1+B)}{\rho_e u_e}\exp\left(\frac{\varkappa\zeta}{B}\int_0^{1+B}\sqrt{\frac{\omega}{\omega_w z}}\,dz\right),$$

where the prime denotes differentiation with respect to z.

We have used the method described above to analyze several cases of ablation of a plate and a sphere of radius 2 m, made from a plastic material similar in composition to textolite [7]. At a temperature of about 2000°K and a pressure on the order of one atmosphere, as shown in [7], the following components are mainly present on an ablating surface: N_2, CO, H_2, HCN, C_2H_2, and the most pronounced reaction is

$$C_2H_2+N_2\rightleftarrows 2HCN.$$

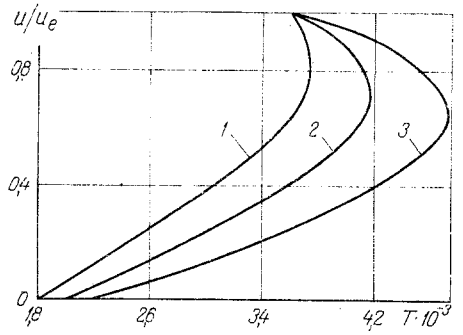

Fig. 1. Temperature profiles, T, °K, in a boundary layer on a plate for three values of the Mach number: 1) M_e = 3.06; 2) 3.78; 3) 4.50.

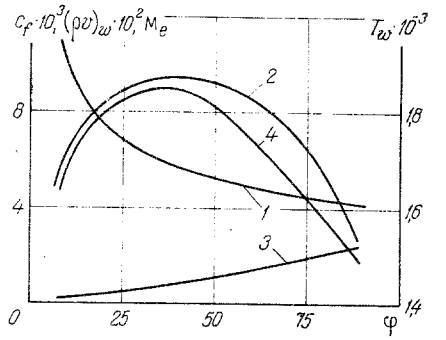

Fig. 2. Characteristics of a turbulent boundary layer on a sphere versus angular coordinate φ, deg 1) c; 2) T_w, °K; 3) M_e; 4) $(\rho v)_w$, kg/m²sec.

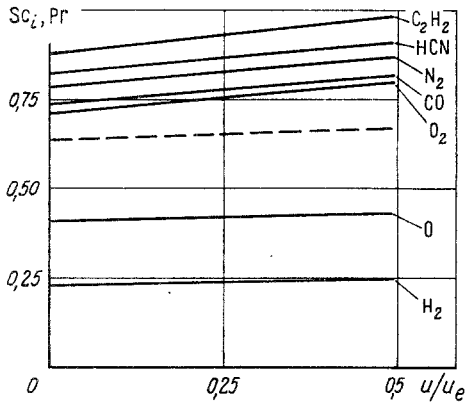

Fig. 3. Distributions of Schmidt (solid curves) and Prandtl (dashed) numbers in the laminar sublayer.

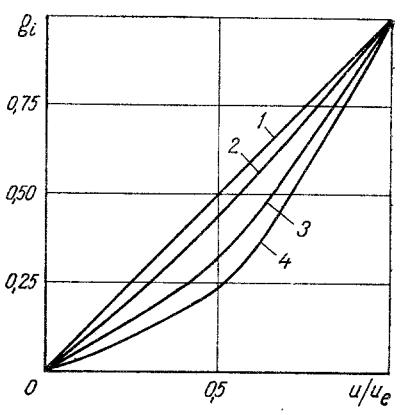

Fig. 4. Profiles of relative concentrations $g_i = (c_i - c_{iw})/(c_{ie} - c_{iw})$ in the boundary layer: 1) case Sc = 1; 2) O_2; 3) O; 4) H_2.

Two components are added in the boundary layer: O, O_2, or O, N, depending on the flow conditions. Thus, in the given model the gas in the boundary layer is assumed to consist of seven components.

Typical results of the computations for ablation of a plate and one case of ablation of a sphere (M_∞ = 16, p_{e0} = 1 atm, T_{e0} = 6000°K) are given in Figs. 1-4. In computing the parameters at the outer edge of the boundary layer, we consider the velocity gradient of the outer flow to be constant over the surface of the sphere and equal to its value at the critical point.

Figure 1 gives the temperature profiles on a plate for three values of the freestream Mach number. With an increase in the latter, both the surface temperature and the maximum temperature in the boundary layer are observed to increase.

The distributions of the surface temperature T_w, mass loss rate $(\rho v)_w$, and coefficient of friction c_f on a sphere as a function of the angular coordinate φ are given in Fig. 2. Also shown there is the variation of the Mach number at the outer edge of the layer. The maximum values of T_w and $(\rho v)_w$ occur in the vicinity of the sonic point, consistent with the published experimental data on turbulent boundary layers [5].

Figure 3 gives the distributions of the effective Schmidt numbers Sc_i and Prandtl number Pr in the laminary sublayer. We note that Sc_i and Pr vary only slightly within the bounds of the laminar sublayer (about 5 to 10%); for this reason they are often assumed to be constant in the sublayer and equal to the surface values, without risking appreciable error in the final results. The effective Schmidt numbers decrease with the molecular weight and differ very little for components having close molecular weights, so that it is sometimes possible to decrease the number of components by clustering them in the analysis of diffusion.

Profiles of the relative concentrations of three components having different effective Schmidt numbers are given in Fig. 4. Also shown is the concentration profile for the special case Sc = 1. The curves tend to straighten with increasing Schmidt numbers, approaching the curve for Sc = 1.

NOTATION

x, y, coordinates along the surface and along the normal to it; u, v, longitudinal and transverse velocity components; m, c_p, molecular weight and heat capacity of gaseous mixture; r_w, radius of transverse curvature of body; p, ρ, T, pressure, density, and temperature of gas; μ, λ, molecular-viscosity and thermal-conductivity coefficients; m_i, c_i, c_{pi}, I_i, D_i, molecular weight, mass concentration, heat capacity, diffusion flux, and effective diffusivity of i-th component; τ, tangential frictional stress; ε, λ_T, D_T, turbulent viscosity, thermal conductivity, and diffusivity; N, l, numbers of chemical components and elements; ν_{ij}, stoichiometric coefficient; r_{ki}, fraction of k-th element in i-th component; h, enthalpy; h_{sw}, enthalpy of material heated to surface temperature; $h_{-\infty}$, enthalpy of unheated material; $h^{(1)}$, enthalpy of condensed phase; $c_i^{(1)}$, concentration of i-th component in the condensed phase; Δ, heat of phase transition; K_{pi}, equilibrium constant of chemical reaction; σ, Stefan–Boltzmann constant; γ, surface emissivity; ξ, z, modified Crocco variables; H, shape parameter; $B = [2(\rho v)_w]/(c_f \rho_e u_e)$, injection parameter; $c_f = (2\tau_w)/(\rho_e u_e^2)$, friction coefficient; $\zeta = (2\rho_w/c_f\rho_e)^{1/2}$, friction parameter; M, Sc, Pr, Mach, Schmidt, and Prandtl numbers; δ^*, δ^{**}, boundary-layer and momentum-loss thicknesses; α, $\varkappa$, turbulence constants, Indices: e, outer edge of boundary layer; w, surface of body; t, transition point; L, boundary of laminar sublayer.

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